SECURITY CLASSIFICATION OF THIS PAGE

UNCLASSIFIED

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION: UNCLASSIFIED

1b. RESTRICTIVE MARKINGS

2a. SECURITY CLASSIFICATION AUTHORITY

2b. DECLASSIFICATION / DOWNGRADING SCHEDULE

3. DISTRIBUTION / AVAILABILITY OF REPORT: Approved for public release; distribution unlimited.

4. PERFORMING ORGANIZATION REPORT NUMBER(S): AD-A213 630

5. MONITORING ORGANIZATION REPORT NUMBER(S): AFOSR-TR- 89-1274

6a. NAME OF PERFORMING ORGANIZATION: Center for the Mathematical Sciences

6b. OFFICE SYMBOL (If applicable)

7a. NAME OF MONITORING ORGANIZATION: AFOSR/NM

DTIC ELECTE OCT 24 1989

6c. ADDRESS (City, State, and ZIP Code): University of Wisconsin-Madison, 610 Walnut Street, Madison, WI 53705

7b. ADDRESS (City, State, and ZIP Code): AFOSR/NM, Bldg 410, Bolling AFB DC 20332-6448

8a. NAME OF FUNDING / SPONSORING ORGANIZATION: AFOSR

8b. OFFICE SYMBOL (If applicable): NM

9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER: AFOSR-87-0191

8c. ADDRESS (City, State, and ZIP Code): AFOSR/NM, Bldg 410, Bolling AFB DC 20332-6448

10. SOURCE OF FUNDING NUMBERS

PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT ACCESSION NO.
61102F	2304	A9	

11. TITLE (Include Security Classification): Interdisciplinary Research in Viscoelasticity & Rheology

12. PERSONAL AUTHOR(S): David S. Malkus and John A. Nohel

13a. TYPE OF REPORT: Final

13b. TIME COVERED: FROM 5/87 TO 5/89

14. DATE OF REPORT (Year, Month, Day): August 25, 1989

15. PAGE COUNT: 13

16. SUPPLEMENTARY NOTATION

17. COSATI CODES

FIELD	GROUP	SUB-GROUP

18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

A deep understanding of viscoelasticity and rheology is crucial to advanced materials engineering and process design. Examples of such advanced materials are high-strength polymers and additives for lubricants; process design problems include spinning of synthetic fibers and injection molding. The materials involved in these technologies are often highly elastic and very viscous. As a consequence, they often display behavior intermediate between that of a solid and that of a fluid, and their dynamic response involves multiple time-scales. Our goal is to understand the predictions of equations of motion coupled with various constitutive assumptions for advanced, complex materials and to put this knowledge to use in modelling, the design of algorithms, and the computational solution of practical problems. To achieve our goals, our interdisciplinary program has adapted and extended tools in nonlinear analysis of partial differential equations, analytical and computational techniques for hyperbolic conservation laws, and computational techniques from nonlinear structural dynamics. Awareness of the latest developments in the rheology of viscoelastic liquids was vital in these efforts. (JES)

20. DISTRIBUTION / AVAILABILITY OF ABSTRACT: ☐ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT ☐ DTIC USERS

21. ABSTRACT SECURITY CLASSIFICATION

22a. NAME OF RESPONSIBLE INDIVIDUAL: Dr. Arje Nachman

22b. TELEPHONE (Include Area Code): (202) 767-4939

22c. OFFICE SYMBOL: NM

DD FORM 1473, 84 MAR — 83 APR edition may be used until exhausted. All other editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE: UNCLASSIFIED

89 10 24 085

AFOSR-TR- 89-1274

INTERDISCIPLINARY RESEARCH IN VISCOELASTICITY & RHEOLOGY*

David S. Malkus & John A. Nohel

with

Deborah Brandon, Robert C. Rogers, Athanassios E. Tzavaras & Students

Center for Mathematical Sciences
University of Wisconsin - Madison
Madison, WI 53705

A-1

1 September 1989

Final Technical Report for Grant AFOSR 87-0191 Covering Period 1 May 1987 to 31 May 1989

DISTRIBUTION: unclassified/unlimited

Prepared for

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH
Directorate of Mathematical & Information Sciences
Bolling AFB DC 20332-6448

* The views and conclusions contained in this document are those of the authors and should not be interpreted as necessarily representing official policies or endorsements, either expressed or implied, of the Air Force Office of Scientific Research or the US Goverment.

INTERDISCIPLINARY RESEARCH ON VISCOELASTICITY AND RHEOLOGY

TABLE OF CONTENTS

A-1

1. Research Objectives

A deep understanding of viscoelasticity and rheology is crucial to advanced materials engineering and process design. Examples of such advanced materials are high-strength polymers and additives for lubricants: process design problems include spinning of synthetic fibers and injection molding. The materials involved in these technologies are often highly elastic and very viscous. As a consequence, they often display behavior intermediate between that of a solid and that of a fluid, and their dynamic response involves multiple time-scales. The understanding of the equations of motion coupled with various constitutive assumptions at the mathematical level is crucial for modelling, design of algorithms and computational solution of practical problems. The goals of our research can be summarized as follows:

1. To understand qualitative properties of the mathematical model: the global existence and uniqueness of solutions, dependence on data, regularity and asymptotic behavior of solutions for large time, approach to steady states, etc.
2. To understand the physical consequences of the model. Do any or all of the possible solutions make physical sense? Do solutions that have mathematically interesting character correspond to observed phenomena? Do they predict behavior that should be studied in the laboratory? What solutions to the problem are relevant to processing and design?
3. To understand the physical model. How do the observed solutions correspond to the molecular or continuum model on which they are based? Can the character of these solutions serve to validate the physical model or suggest improvements in it?
4. To design numerical methods that account for the mathematics and reproduce the physics. The current focus is on one-dimensional model problems, but in the long-term, the goal is to adapt results for the model problems to flows in complex geometries.
5. To study the broad mathematical implications of 1. - 4. for these and related classes of problems. We note that concepts such as weak solutions, shocks, shear bands, phase changes, etc arise in a variety of problems of physical and technological importance.

2. Research Accomplishments of Principal Investigators

In recent joint work with B. J. Plohr [10 – 13, 24, 25], we investigated several models with these features. We discovered several dramatic new phenomena in shear flow that appear to be relevant to material instabilities that disrupt polymer processes.

One striking phenomenon has been observed by Vinogradov *et al.* (1972*) in the flow of monodisperse polyisoprenes through capillaries. They found that the volumetric flow rate increased dramatically at a critical stress that was independent of molecular weight. This phenomenon, which is called "spurt," had been overlooked or dismissed by rheologists because no plausible mechanism was known to explain it in the context of steady flows. Spurt was lumped together with instabilities such as "slip," "apparent slip," and "melt fracture," which are poorly understood (see recent surveys by M. Denn (1989) and J. Pearson (1985)). While regarded as anomalous, these instabilities can severely disrupt

* References identified by date are listed in chronological order in Sec. 5

polymer processes; they can be avoided in practice only with *ad hoc* engineering expedients. The mechanisms of such phenomena were not understood because the governing equations are analytically intractable, and because popular numerical methods for steady viscoelastic fluid flows falter in this "high Weissenberg number" regime, and thus cannot model the spurt phenomenon.

By contrast, our recent work shows that satisfactory modelling and explanation of spurt and related phenomena require studying the full dynamics of the equations of motion and constitutive equations. The common and key feature of constitutive models that exhibit spurt and related phenomena is a non-monotonic relation between the steady shear stress and strain rate. This allows jumps in the steady strain rate to form when the driving pressure gradient exceeds a critical value; such jumps correspond to the sudden increase in volumetric flow rate observed in the experiments of Vinogradov, *et al.* (1972). The governing systems used to model such one-dimensional flows are analyzed in [6, 10 – 12] by numerical techniques and simulation, and in our more recent work by analytical methods. The systems derive from fully three-dimensional differential constitutive relations with m-relaxation times (based on work of Johnson and Segalman (1977) and Oldroyd (1958)). They are evolutionary, globally well posed, and they possess discontinuous steady states of the type mentioned above that lead to an explanation of new phenomena.

We model these flows by decomposing the total shear stress into a polymer contribution, evolving in accordance with a differential constitutive relation with a single relaxation time and a Newtonian viscosity contribution. The flows can also be modelled by a system based on a differential constitutive law with two widely spaced relaxation times but no Newtonian viscosity contribution. We developed three numerical methods for the model initial boundary value problem: one based on Plohr's recent work on Riemann problems for hyperbolic conservation laws with a regularizing Newtonian viscosity, another based on using the total stress (rather than the velocity gradient) as a dependent variable in the balance of linear momentum, and a third based on appropriate modifications of an existing solid-mechanics code. Each of the three techniques yields satisfactory qualitative and quantitative agreement with Vinogradov's experimental results. Numerical simulation [6, 10 – 12] of transient flows at high Weissenberg (Deborah) number and very low Reynolds number exhibited spurt, shape memory, and hysteresis; furthermore, it predicted other effects, such as latency, normal stress oscillations, and molecular weight dependence of hysteresis, that should be analysed further and tested in rheological experiment.

Our numerical work suggested a scaling valid for Vinogradov's material which allowed the governing system of partial differential equations to be reduced to a pair of nonlinear ordinary differential equations by setting a key parameter (ratio of Reynolds number to Deborah number which is of order 10^{-12} for a Vinogradov material) to zero. The resulting dynamics can be determined completely. The results of the phase plane analysis are summarized in [13]; analytical results will appear in [24, 25]. At the mathematical level, it remains a challenge (being addressed) to resolve the singular perturbation problem arising in this situation between the full and reduced systems. In [25], we also justify the use of Newtonian viscosity to model the effects of relaxation times which are very much shorter than the fundamental time constant of the macromolecules. We determine conditions under which the character of the phase portrait of a more realistic, two relaxation-time model

without Newtonian viscosity matches matches the phase portrait of the simpler model with Newtonian viscosity. Our analytic results helped us to understand certain rather dramatic effects that were observed in the numerical solutions. Moreover, they enabled us to identify the parametric dependence of these effects and to predict their experimental signature.

The analytical results discussed above have motivated Nohel (in collaboration with A. Tzavaras and R. Pego) to undertake a deeper analytical study of the full unsteady shear-flow problem keeping the inertial term. In [26, 27], we discuss results on the nonlinear stability of discontinuous steady states of a model initial-boundary value problem in one space dimension for incompressible, isothermal shear flow of a non-Newtonian fluid between parallel plates, driven by a constant pressure gradient. The model incorporates several properties of the more complex problem described above. The non-Newtonian contribution to the shear stress is assumed to satisfy a simple differential constitutive law. Again, the key feature is a non-monotone relation between the total steady shear stress and steady shear strain rate that results in steady states having, in general, discontinuities in the strain rate. We explain why every solution tends to a steady state as $t \to \infty$, and we identify steady states that are stable. In current work, Nohel, Pego and Tzavaras attempting to extend these results to the full problem discussed above. Preliminary research indicates that discontinuous steady states of the more complex problem are stable (perhaps not asymptotically) if the ratio of Reynolds number to Deborah number is sufficiently small.

To achieve our goals, our interdisciplinary program has adapted and extended tools in nonlinear analysis of partial differential equations, analytical and computational techniques for hyperbolic conservation laws, and computational techniques from nonlinear structural dynamics. Awareness of the latest developments in the rheology of viscoelastic liquids was vital in these efforts.

3. Research Accomplishments of Associates & Students

(A) A. Tzavaras, Van Vleck Assistant Professor. Shear instabilities in the form of shear bands are often observed when metals are deformed at high strain rates. Since shear bands diminish the strength of materials and are often precursors to rupture, their understanding is critical for the development of improved materials. According to a popular theory, shear band formation is the result of a destabilizing feedback mechanism induced by thermal softening properties of materials. At high strain rates, non-uniform straining induces non-uniform heating which, in turn, enhances the plastic flow at hotter regions and reduces it at colder regions. This creates a destabilizing feedback mechanism to which there is opposition from internal dissipation and from strain hardening properties of the material. In order to assess the contribution of the above factors we have been studying the adiabatic plastic shearing of an infinite plate subjected to prescribed tractions or steady shearing at the boundaries. For constitutive law we choose a power law with parameters measuring the relative weight of thermal softening, strain hardenining and strain rate sensitivity. The analysis of the above problem leads to the study of a system of partial differential equations consisting of a parabolic equation coupled through the diffusion coefficient with two equations of hyperbolic type. The main question is to study whether the solution stabilizes at large times or whether nonuniformities develop and the material exhibits an unstable response. This question is studied in [34] with the following results. In

case the shearing deformation is caused by prescribed tractions the parameter space can be decomposed into two subregions. In one of them the solutions are asymptotically attracted to a "uniform" dynamic solution, while in the complement instabilities develop. The analysis indicates that the latter are associated with a collapse of the ability of the material to diffuse the applied stress.

In a joint program with M. Slemrod we are studying the wave fan admissibility criterion for the solution of the Riemann problem for hyperbolic conservation laws. It is well known that in solving the Riemann problem one encounters a loss of uniqueness that has to be accounted for by choosing an appropriate admissibility criterion. According to the wave fan admissibility criterion the admissible solutions are chosen as limits of appropriate "viscosity regularized" problems, that are rigged so as to preserve the invariance of the equations under rescaling of the independent variables. This approach has been succesfully tested for the Riemann problem for the equations of gas dynamics in one space dimension in Eulerian coordinates [32].

(B) **Deborah Brandon,** Post-Doctoral Research Associate. The research involves the modeling and analysis of problems in heat conduction in one dimensional materials with memory. During the reporting period, work was completed on the construction of concrete integral models for such materials, based on a very general theory developed by Gurtin and Pipkin which is compatible with the second law of thermodynamics but which leaves the form of the constitutive functionals for the heat flux and for the internal energy unspecified. The results will appear in [3]. This theory which ignores mechanistic effects predicts a finite speed of propagation for thermal disturbances. However, Gurtin and Pipkin did not provide any concrete examples. Her principal achievement is the explicit construction of constitutive functionals for the heat flux and internal energy which depend on athe temporal history of the temperature gradient and possibly on the present value as well as on the history of the temperature and which also satisfy the necessary and sufficient conditions of the Gurtin-Pipkin theory. These functionals are motivated by models used in viscoelasticity. The key difficulty is that the crucial condition which must be understood deeply and then properly interpreted is in the form of a functional differential (Clausius-Duhem) inequality. As a consequence, she shows that compatibility with the second law requires the internal energy (as well as the heat flux) to have a nontrivial dependence on the history of the temperature gradient as well as on the present value and the history of temperature.

(C) **R. C. Rogers,** Van Vleck Assistant Professor. In addition to the joint work with Nohel and Tzavaras [15] described above, Rogers persued research on self- effect problems in electro-magneto-elasticity [17, 21], modelling ferromagnetic materials [18], and problems in compensated compactness and weak continuity [19, 20].

(D) **K. Barki,** Ph.D. student. Mr. Barki's work [35] is part of a project involving Malkus, Barki, Y.-C. Tsai, and another student, R. Cornwell (who is supported by AFOSR funds from a different grant, AFOSR 89-0220). The purpose of the project is to develop a dynamic finite element code for the solution of problems in solid thermo-viscoplasticity and the use of this code to investigate the calibration and resulting resolution of laboratory stress-analysis devices based on differential thermography. Malkus is collaborating with

M. Plesha, R. Rowlands, and B. Sandor (Dept. Engineering Mechs., U.-W.) on a new device, called SPATE, that promises to be a powerful tool in experimental stress analysis. The device operates by sensing and interpreting small temperature differences induced by cyclic loading. At present, the device has several operating restrictions: first, the theory behind it applies only to linearly elastic materials; second, the output is the trace of the stress tensor, not individual stress components; and third, this stress invariant can be sensed reliably only at the surface of the test specimen. To make SPATE a more flexible tool in stress analysis, it is important (i) to determine principal stress components from the data for the stress invariant and (ii) to use the separated surface stress distributions to deduce interior stress distributions. Solving these problems requires an accurate model of material behavior and appropriate boundary conditions, so that the governing equations may be formulated and solved.

Malkus and Cornwell have successfully simulated thermographic signals emitted by a viscoelastic body in uniaxial, cyclic loading by developing algorithms for coupled thermo-viscoelasticity, including dissipation heating. Extensions to include materials with yield stresses are in progress. We are now developing the capability to solve thermally-dependent problems in a higher dimensional setting, based on the work of K. Barki. His thesis research involves the development of new transient algorithms for finite element analysis of viscoplastic materials. The major focus is on a class of implicit/explicit methods with origins in structural dynamics, generalizing the method developed by Malkus to solve dynamic spurt. There are two fundamental ideas: first to allow implicit treatment of the elastic (highest) wave speed, which avoids severe stability limits; second, to avoid the need for stress interpolations by advancing the constitutive equations at Gauss-points in ODE fashion. Mr. Barki is focussing on problems in solid, 3-D viscoplasticity in a Lagrangian frame, but the ideas seem to be useful for fluid flow problems in an Eulerian frame. This idea is being pursued by Malkus. Mr. Barki has been using the CMS VAX system and the SDSC Cray XMP/48. He expects his Ph.D. in the fall of 1989.

(E) Y.-C. Tsai, Ph.D. Candidate. Mr. Tsai has been investigating the theoretical mechanics of thermally coupled viscoplasticity and the experimental determination of model parameters: of particular interest is the relaxation spectrum of the heat capacity term in the energy equation. The next phase of the planned research is to investigate the consequence of viscoplastic behavior on differential thermography, by simulating the SPATE stress analysis system. The theory behind differential thermography only rigorously applies to linearly elastic material, but material failure mechanisms often involve plastic deformation, stress relaxation, and creep. How such mechanisms would be sensed by differential thermography: we are addressing the question of how the device could be calibrated, and what the accuracy of such measurements would be.

4. Publications

A. BOOKS

1. R. Cook, D. S. Malkus, and M. E. Plesha, Concepts and Applications of Finite Element Analysis, Third Edition, John Wiley and Sons, New York, 1989. 630 pp.
2. M. Renardy, W.J. Hrusa, and J.A. Nohel, Mathematical Problems in Viscoelasticity, *Pitman Monographs and Surveys in Pure and Applied Mathematics*, vol. 35, Longman

Scientific & Technical, Essex, England (1987), 273 pp

B. PAPERS PUBLISHED OR ACCEPTED

3. D. Brandon and W. J. Hrusa, Construction of a class of integral models for heat flow in materials with memory, *Journal of Integral Equations*, accepted.
4. A. Friedman and A. E. Tzavaras, Combustion in a porous medium, *SIAM J. Math. Anal.* 19 (1988), 509–519.
5. W. J. Hrusa, J. A. Nohel and M. Renardy, Initial value problems in viscoelasticity, *Applied Mechanics Reviews* **41** (1988), 371-378.
6. R. W. Kolkka, D. S. Malkus, M. G. Hansen, G. R. Ierley, and R. A. Worthing, Spurt phenomena of the Johnson Model fluid and related models, *J. Non-Newtonian Fluid Mechanics* **29** (1988), 303-325.
7. D. S. Malkus and M. F. Webster, On the accuracy of finite element and finite difference predictions of non-Newtonian slot pressures for a Maxwell fluid, *J. Non-Newtonian Fluid Mechanics* 25 (1987), 93–127.
8. D. S. Malkus and X. Qiu, Divisor structure of finite element eigenproblems arising from negative and zero masses, *Comput. Meths. Appl. Mech. Engrg.* 66 (1988), 365–368.
9. D. S. Malkus, M. E. Plesha, and M.-R. Liu, Reversed stability conditions in transient finite element analysis, *Comput. Meths. Appl. Mech. Engrg.* 68 (1988), 97–114.
10. D. S. Malkus, J. A. Nohel, and B. J. Plohr, Time-dependent shear flow of a non-Newtonian fluid (preliminary report), *Transactions of the Sixth Army Conference on Applied Mathematics and Computing (Boulder, 1988).*
11. D. S. Malkus, J. A. Nohel, and B. J. Plohr, Time-dependent shear flow of a non-Newtonian fluid, *Current Problems in Hyberbolic Problems: Riemann Problems and Computations (Bowdoin,1988)*, Contemporary Mathematics, Amer. Math. Soc., ed. B. Lindquist, accepted.
12. D. S. Malkus, J. A. Nohel, and B. J. Plohr, Dynamics of shear flow of a non-Newtonian fluid, *Journal of Computational Physics*, accepted.
13. D. S. Malkus, J. A. Nohel, and B. J. Plohr, Analysis of spurt phenomena for a non-Newtonian fluid, *Proceedings of Conference on Problems that Change Type (Stuttgart, October (1988)*, Springer Verlag Lecture Notes, accepted.
14. J.A. Nohel and M. Renardy, Development of singularities in nonlinear viscoelasticity, Proceedings of Workshop on Amorphous Polymers, *IMA Volumes in Mathematics and its Applications*, Vol. 6, Springer-Verlag (1987), 139–152.
15. J.A. Nohel, R.C. Rogers, and A. Tzavaras, Weak solutions for a nonlinear system in viscoelasticity, *Comm. in P.D.E.* 13 (1988), 97–127.
16. J.A. Nohel, R.C. Rogers, and A.E. Tzavaras, Hyperbolic conservation laws in viscoelasticity, in: Volterra Integro Differential Equations in Banach Spaces and Applications, G. da Prato & M. Ianelli (Eds), Longman Scientific and Technical (in press).
17. Robert C. Rogers and Stuart S. Antman, Steady-state problems of nonlinear electro-magneto-thermo-elasticity, *Arch. Rat. Mech. Anal.* 95 (1986), 279–323.
18. Robert C. Rogers, Nonlocal problems in electromagnetism, in Stuart S. Antman, J.L. Ericksen, David Kinderlehrer, and Ingo Müller, editors, *Metastability and Incompletely Posed Problems*, IMA, Springer-Verlag, New York (1986).

19. Joel W. Robbin, Robert C. Rogers, and Blake Temple, On weak continuity and the Hodge decomposition, *Transactions of the AMS* 302 (1987).
20. Robert C. Rogers and Blake Temple, A sufficient condition for weak continuity of polynomials in the method of compensated compactness, *Transactions of the AMS*, accepted.
21. Robert C. Rogers, Nonlocal variational problems in nonlinear electro-magneto-elasticity, *SIAM J. of Math. Analysis*, accepted.
22. A. E. Tzavaras, Effect of thermal softening in shearing of strain rate dependent materials, *Arch. Rational Mech. and Anal.* 99 (1987), 349–374.
23. M. Yao and D. S. Malkus, Boundary node correction and super-convergence in the finite element method, preprint (1989), *Int. J. Num. Meth. Fluids*, accepted.

C. PAPERS SUBMITTED

24. D. S. Malkus, J. A. Nohel, and B. J. Plohr, Analysis of new phenomena in shear flow of non-Newtonian fluids, preprint (1989), *Siam. J. Appl. Math.*, submitted.
25. D. S. Malkus, J. A. Nohel, and B. J. Plohr, Quadratic dynamical systems describing phenomena in shear flow of non-Newtonian fluids, preprint (1989), *Proceedings IMA Workshop on Nonlinear Evolution Equations That Change Type (1989)*, Springer Verlag Lecture Notes, submitted.
26. J. A. Nohel, R. Pego, and A. E. Tzavaras, Stability of discontinuous steady states in shearing motions of a non-Newtonian fluid, preprint (1989), *Proc. Royal Soc. Edingburgh, Series A.*, submitted.
27. J. A. Nohel, R. Pego, and A. E. Tzavaras, Nonlinear stability in non-Newtonian flows, preprint (1989), *Proceedings IMA Workshop on Nonlinear Waves (1989)*, Springer Verlag Lecture Notes, submitted.

D. PAPERS IN PREPARATION

28. D. Brandon, Global existence and asymptotic stability for a nonlinear integrodifferential equation modeling heat flow (in prepa.ation).
29. D. Brandon and W. J. Hrusa, Global existence of smooth shearing motions of vicoelastic fluids (in preparation).
30. M. W. Johnson and D. S. Malkus, Shear-flow instabilities and numerical algorithms for non-Newtonian flows (in preparation).
31. D. S. Malkus and Y.-C. Tsai, Stability analysis of implicit-explicit time integration for viscoelastic flow (in preparation).
32. D. S. Malkus and Y.-C. Tsai, New algorithms for non-Newtonian Flows (in preparation).
33. M. Slemrod and A.E. Tzavaras, A viscosity approach for the solution of the Riemann problem in Eulerian gas dynamics (in preparation).
34. A.E. Tzavaras, Interplay of thermal softening and strain rate sensitivity on the response of shearing motions (in preparation).

E. THESES

35. K. Barki, Ph. D. Thesis, Dept Engineering Mechanics, University of Wisconsin, Madison (in preparation), abstract follows:

THESIS ABSTRACT

The thesis summarizes the recent development of viscous material models and proposes a viscoelastic-viscoplastic material model which includes a widely used viscoplastic and viscoelastic models as special cases. The model also facilitates the inclusion of a variety of internal variables proposed recently by other researchers. A modified Euler algorithm is used to integrate the constitutive equation. An implicit-explicit predictor corrector scheme is used to solve the transient equations. Derivation of stability conditions for viscoelastic part is illustrated using the central difference method. The development of the general purpose nonlinear program is discussed. The use of supercomputing also is discussed in the thesis, although the thesis concerns itself principally with the material behavior of the model. Numerical examples for one dimensional and plane strain models are presented with a variety of loadings.

5. Additional References

J. Oldroyd, Non-Newtonian effects in steady motion of some idealized elastico-viscous liquids, *Proc. Roy. Soc. London*, A 245 (1958) 278–297.

G. Vinogradov, A. Malkin, Yu. Yanovskii, E. Borisenkova, B. Yarlykov, and G. Berezhnaya, Viscoelastic properties and flow of narrow distribution polybutadienes and polyisoprenes *J. Polymer Sci., Part A-2*, 10 (1972).

M. Johnson and D. Segalman, A Model for viscoelastic fluid behavior which allows non-affine deformation, *J. Non-Newtonian Fluid Mech.*, 2 (1977) 255–270.

J. Pearson, Mechanics of Polymer Processing, Elsevier Applied Science,London (1985).

M. Denn, Issues in viscoelastic fluid dynamics, *Annual Reviews of Fluid Mechanics*, 1989, to appear.

6. Professional Personnel

D. S. Malkus, Professor, Engineering Mechanics & Center for the Math. Sciences

J. A. Nohel, Professor, Mathematics & Center for the Math. Sciences

R. C. Rogers, Van Vleck Assistant Professor, Mathematics & Center for the Math. Sciences

A. E. Tzavaras, Van Vleck Assistant Professor, Mathematics & Center for the Math. Sciences

Deborah Brandon, Postdoctoral Researcher, summer 1988, currently at Institute for Mathematics and Its Applications, University of Minnesota

K. Barki, Ph. D. candidate, Engineering Mechanics

Y.-C. Tsai, Ph. D. candidate, Engineering Mechanics

7. Interactions

The following presentations involving significant research interactions were given during the reporting period:

D. S. Malkus, Type change in non-Newtonian fluids, Fluids Research Oriented Group Seminar, Michigan Technological Institute, Houghton, MI, May, 1987.

D. S. Malkus, Numerical aspects of the High Weissenberg Number Problem, *5th International workshop on Numerical Methods in Non-Newtonian Flows*, Lake Arrowhead, CA, June, 1987.

D. S. Malkus, New transient algorithms for non-Newtonian flows, Mathematics Department Seminar, Illinois Institute of Technology, Nov., 1987.

R. C. Rogers, "Nonlocal materials", MIPAC Seminar, UW-Madison, 12/2/87.

A. E. Tzavaras, Invited lecture based on joint work with Nohel and Rogers, Special Session on "Volterra Integral Equations", combined Midwest and Southeast Differential Equations Conference, Vanderbilt University, Nashville, TN, Oct. 1987.

B. J. Plohr, based on joint work with D. S. Malkus and J. A. Nohel "Instabilities in shear flows of viscoelastic fluids with fading memory", Seminar on PDE's and Continuum Models of Phase Transitions, Nice, France, 1/18-22/88.

D. S. Malkus, "New transient algorithms for non-Newtonian flows", Ill. Inst. of Tech., Math Department seminar, 1/26/88.

D. S. Malkus, A stress calculator for BKZ and similar *single integral constitutive equations*, Annual Meeting of the Society of Rheology, Atlanta, Feb., 1988.

J. A. Nohel, "Mathematical problems in viscoelasticity", Rheology Research Center, UW-Madison, March 4, 1988.

A. E. Tzavaras, Invited lecture based on joint work with Nohel and Rogers, Special Session on "Qualitative Theory of Nonlinear PDE", 843rd. Meeting of the AMS, University of Maryland, College Park, MD, April 1988.

A. E. Tzavaras, "Formation of shear bands", LCDS Seminar in Applied Mathematics, Brown University, April 1988.

D. S. Malkus, based on joint work with J. A. Nohel and B. J. Plohr , "Shearing flows of non-Newtonian fluids", Sixth Army Conference on Applied Math. and Computing, Boulder, CO, May 31, 1988.

A. E. Tzavaras, Invited participant and speaker, meeting on "Hyperbolic Systems of Conservation Laws", Mathematisches Forschungsinstitut, Oberwolfach, West Germany, July 1988.

J. A. Nohel, Seminar, Lefschetz Center for Dynamical Systems, Brown University, July 14, 1988.

J. A. Nohel, Invited lecture, AMS Summer Research Conference on "Current Progress in Hyperbolic Systems: Riemann Problems and Computations", Bowdoin College, Brunswick, Maine, July 16-22, 1988.

J. A. Nohel, Invited lecture, International Conference on "Problems Involving Change of Type", University of Stuttgart, October 11-14, 1988.

A. E. Tzavaras, Seminar, University of Michigan, November, 1988.

J. A. Nohel, Applied Math. Colloquium, Princeton University, December 16, 1988.

J. A. Nohel, Invited participant in three Workshops on nonlinear problems at Institute for Mathematics and Its Applications, University of Minnesota, Winter-Spring, 1989.

A. E. Tzavaras, Invited participant, program on Nonlinear Waves at Institute for Mathematics and Its Applications, University of Minnesota, Winter-Spring, 1989.

A. E. Tzavaras, Two lectures at Division for Applied Mathematics, Brown University, January 1989.

D. S. Malkus, "Implicit-explicit time integration methods for shear flows with spurt instabilities," 60th Annual Meeting of the Society of Rheology, Gainsville, Fla., Feb. 13, 1989.

During the reporting period professional interactions (in some cases joint work) continued with B. J. Plohr (UW-Madison), W. J. Hrusa (CMU), L. Tartar (CMU), M. Slemrod (UW-Madison), C. M. Dafermos (Brown), R. Pego (U. of Michigan), M. Renardy (VPI), D. D. Joseph (U. of Minnesota), E. T. Olsen & B. Bernstein (IIT), T. J. R. Hughes (Stanford), James Walters (BRL), Arthur R. Johnson (Leader, Structural Mechanics Group, Watertown), and members of the Madison Rheology Research Center.